4.2 - Translations of Sine and Cosine

Warmup

Evaluate each expression.

1) $\dfrac{\sin 30° + \cos 60°}{2}$

$\dfrac{1}{2}$

2) $\dfrac{4\sin 300° - 2\cos 30°}{3}$

$-\sqrt{3}$

3) $4(\sin 30°)(\cos 60°)$

1

4) $\sin 30° + \sin 60°$

$\dfrac{1 + \sqrt{3}}{2}$

5) $(\sin 60°)^2 + (\cos 60°)^2$

1

6) $8(\sin 120°)(\cos 120°)$

$-2\sqrt{3}$

4.2 - Translations of Sine and Cosine

Reflections

Recall sine and cosine as odd and even functions.

Odd Function	**Even Function**
$A \sin(-Bx) = -A \sin(Bx)$	$A \cos(-Bx) = A \cos(Bx)$

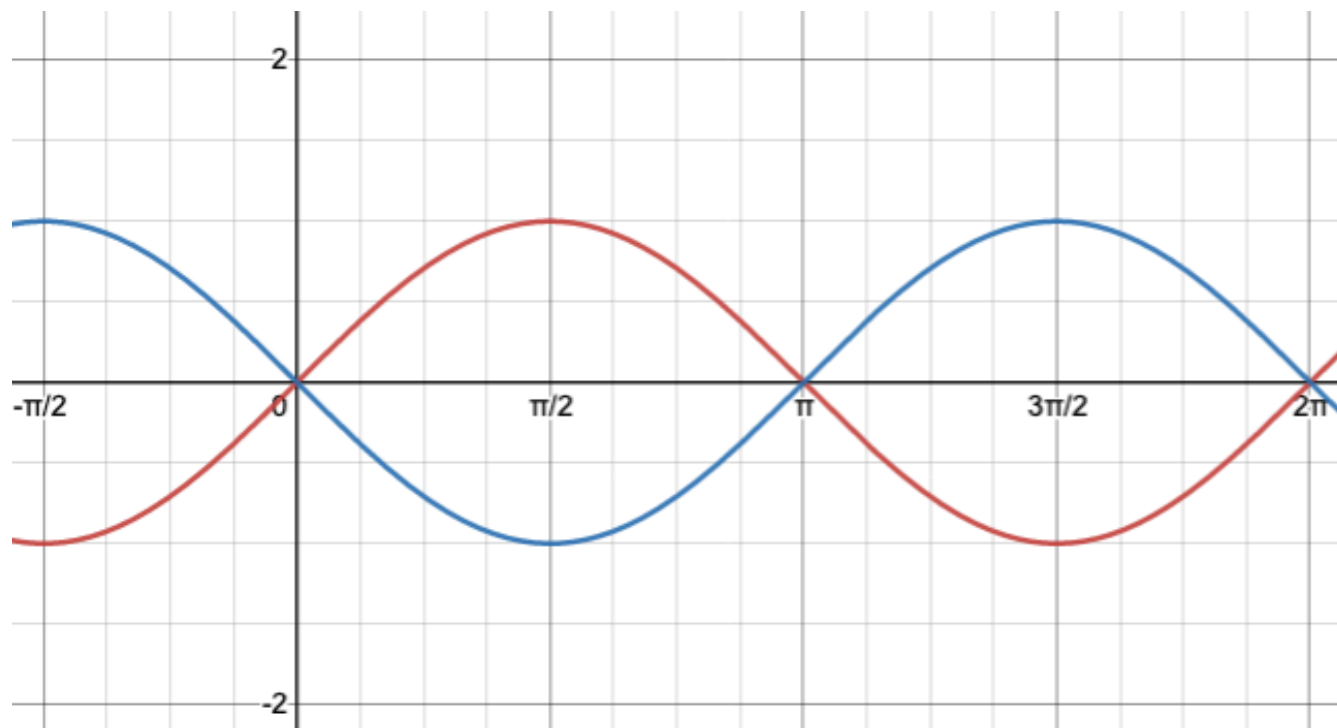**No need to consider negative values of B!!!**
It can be converted to positive/negative values of A.

4.2 - Translations of Sine and Cosine

Reflections

Consider the graphs $y = \sin x$ and $y = -\sin x$

$-f(x)$ is a reflection about the x-axis of $f(x)$
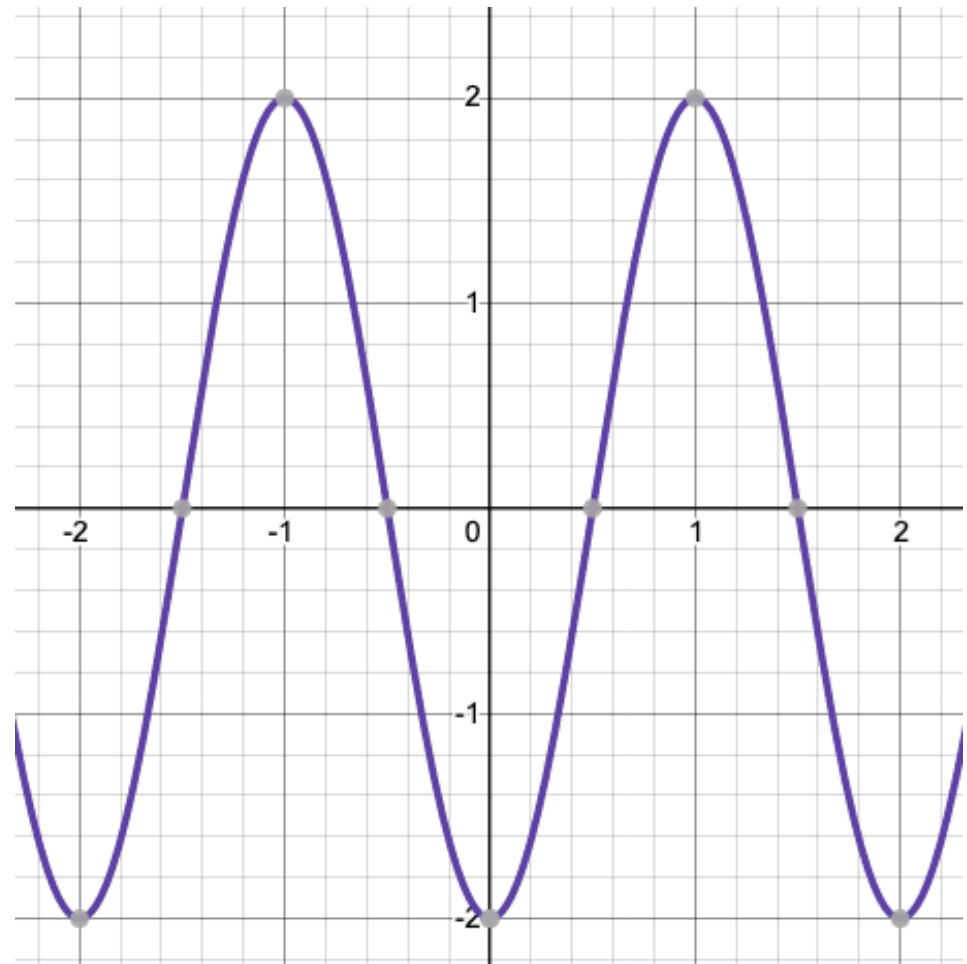
4.2 - Translations of Sine and Cosine

Reflections

Graph $y = -2\cos(\pi x)$

Amplitude $\quad |A| = 2$

Period $\quad\quad p = \dfrac{2\pi}{\pi} = 2$

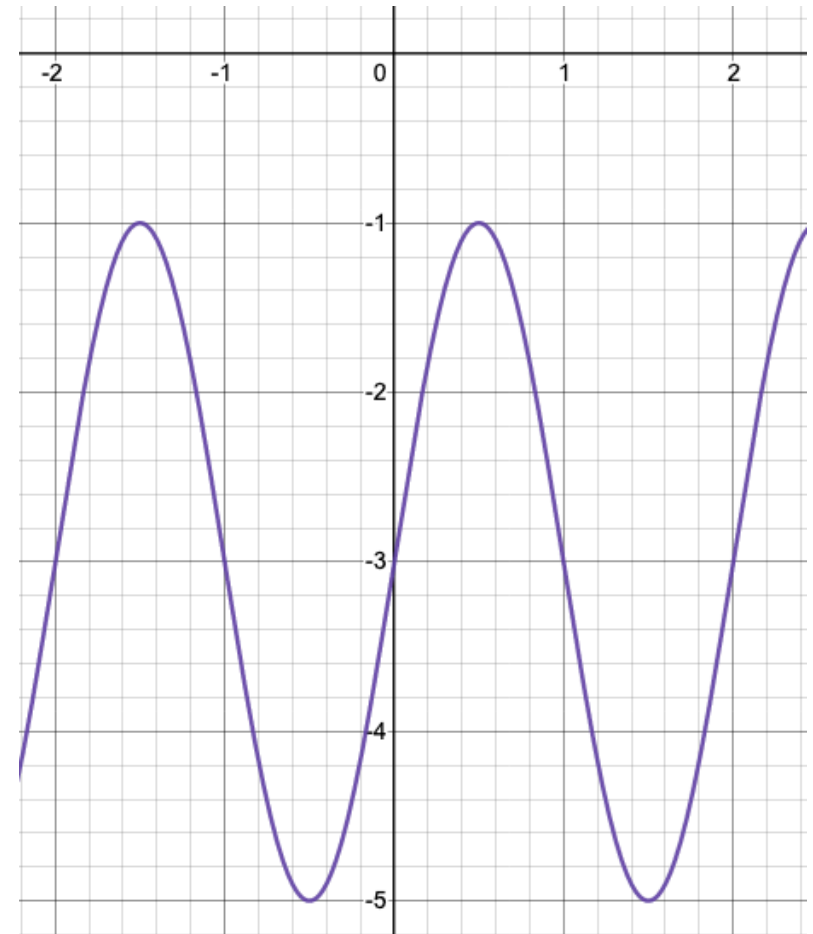# 4.2 - Translations of Sine and Cosine

Vertical Translations

- $f(x) + k$ shifts the graph of $f(x)$ up k units.
- $f(x) - k$ shifts the graph of $f(x)$ down k units.

Graph $y = 2\sin(\pi x) - 3$

Amplitude $|A| = 2$

Period $p = \dfrac{2\pi}{\pi} = 2$
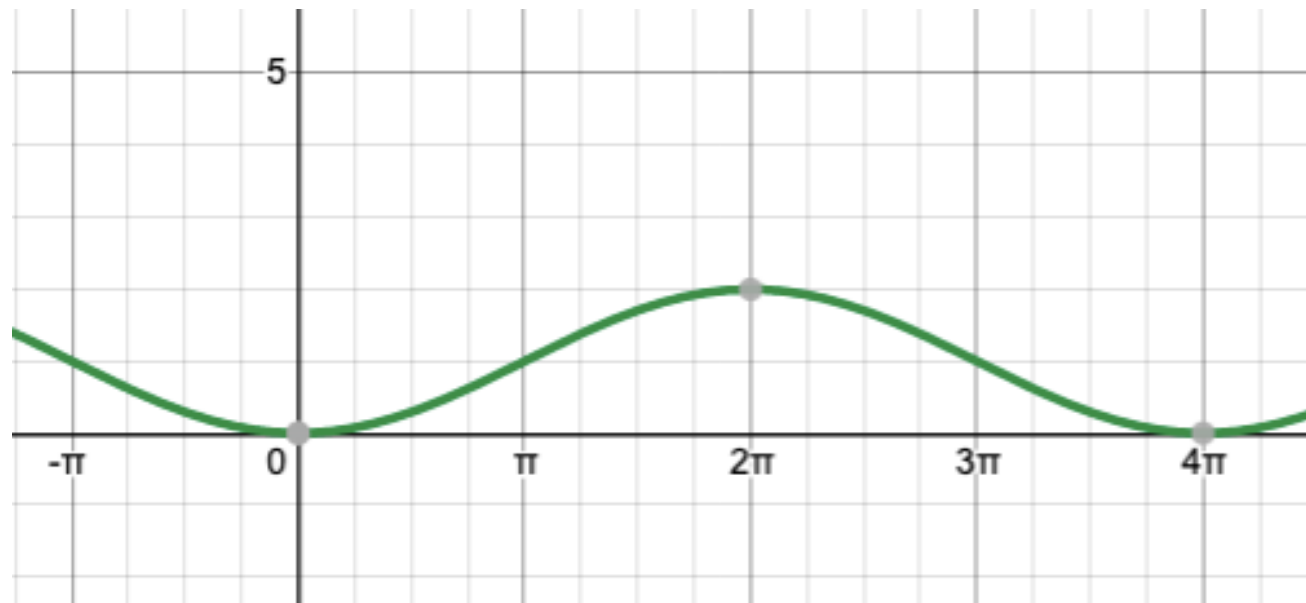
4.2 - Translations of Sine and Cosine

Vertical Translations

Graph $y = -\cos\left(\frac{1}{2}x\right) + 1$

Amplitude $\qquad |A| = 1$

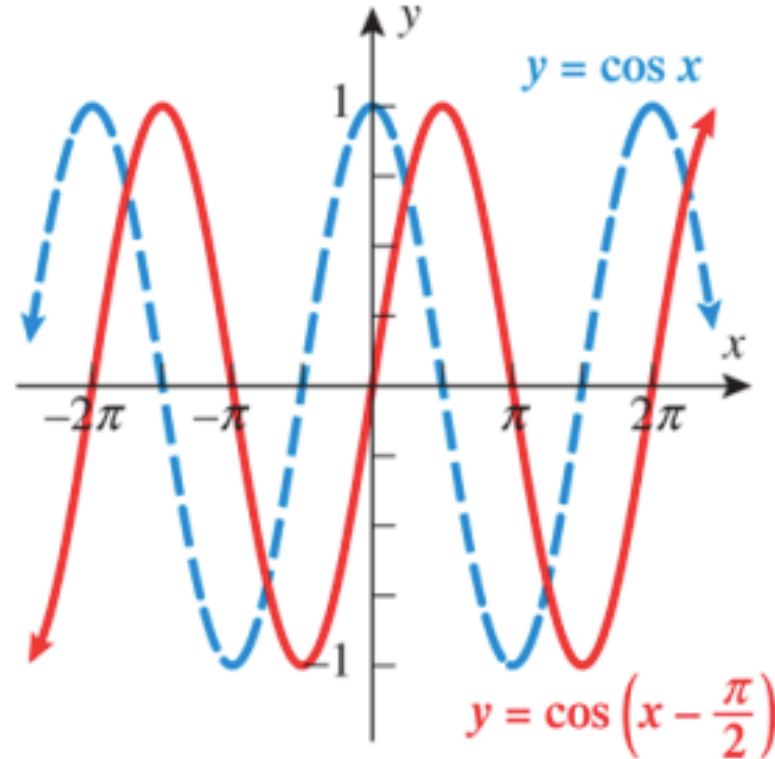Period $\qquad p = \dfrac{2\pi}{\frac{1}{2}} = 4\pi$

4.2 - Translations of Sine and Cosine

Horizontal Translation: Phase Shift

- $f(x + C)$ shifts the graph of $f(x)$ left C units.
- $f(x - C)$ shifts the graph of $f(x)$ right C units.

$y = \cos x$

$y = \cos\left(x - \dfrac{\pi}{2}\right)$

Note that $\cos\left(x - \dfrac{\pi}{2}\right) = \sin x$, so we say they are $\dfrac{\pi}{2}$ out of phase.

4.2 - Translations of Sine and Cosine

Horizontal Translation: Phase Shift
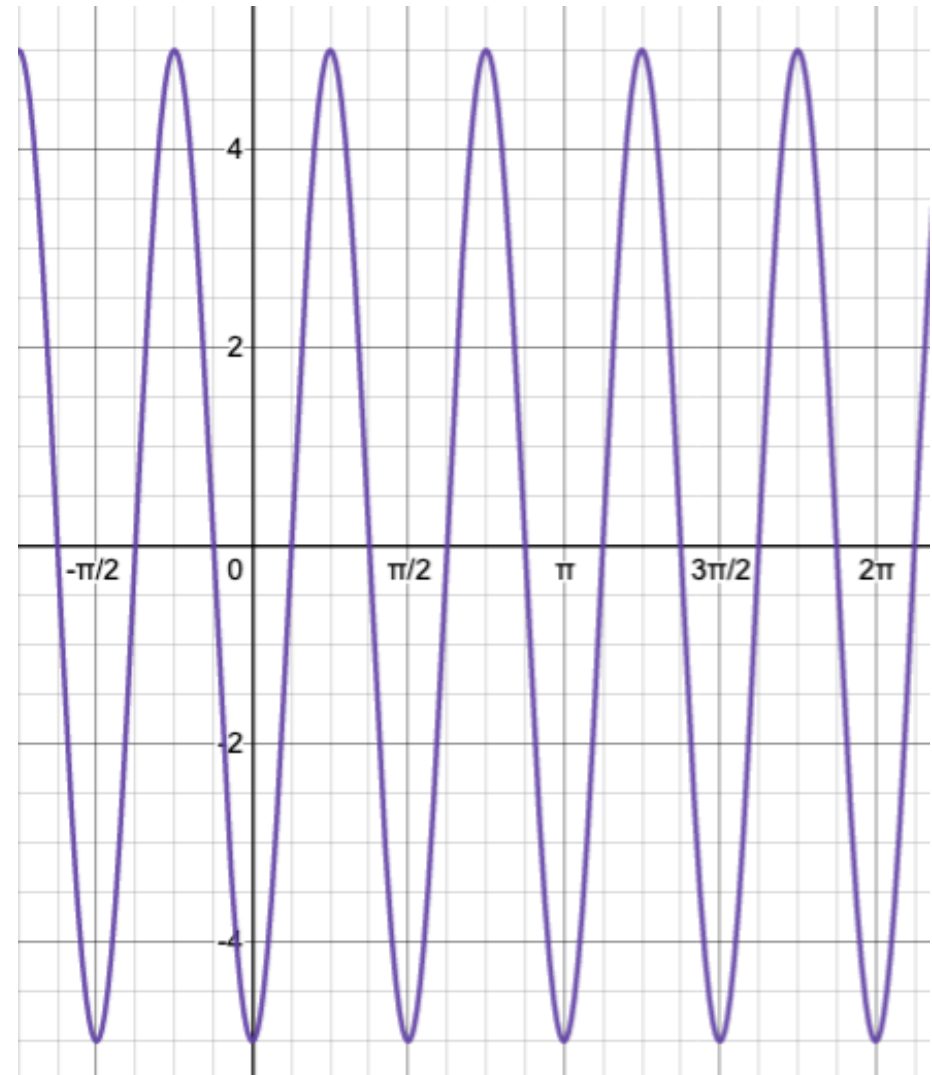
Graph $y = 5\cos(4x + \pi)$

Amplitude $|A| = 5$

Period $p = \dfrac{2\pi}{4} = \dfrac{\pi}{2}$

Phase Shift

$4x + \pi = 0$ to $4x + \pi = 2\pi$

$x = -\dfrac{\pi}{4}$ $x = \dfrac{\pi}{4}$

4.2 - Translations of Sine and Cosine

Horizontal Translation: Phase Shift
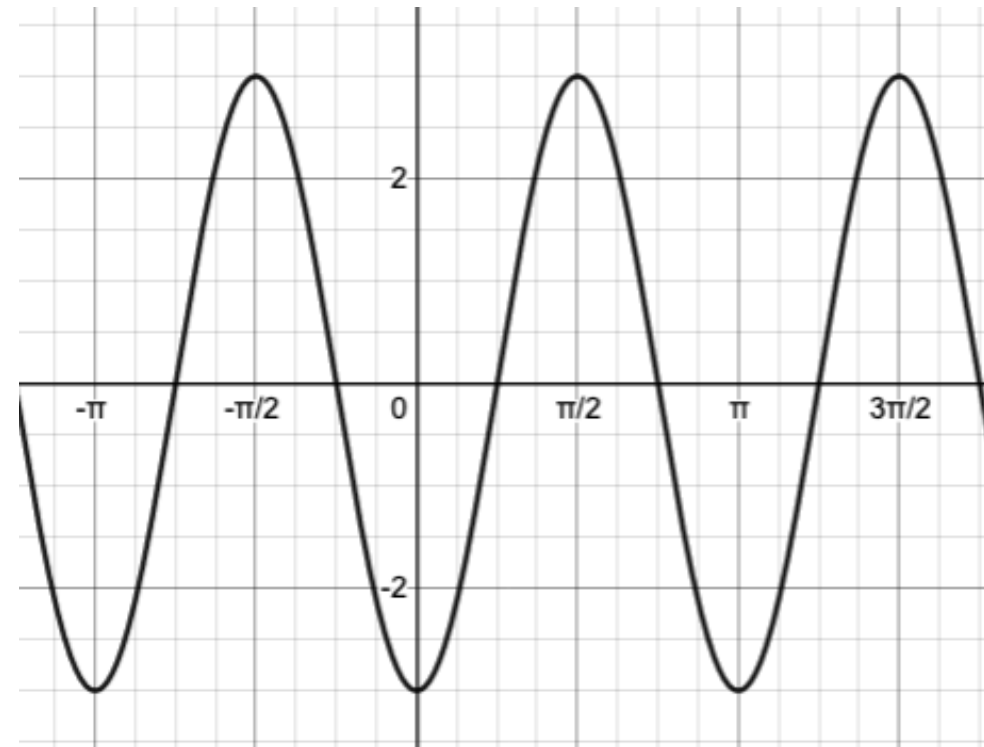
Graph $y = 3\cos(2x - \pi)$

Amplitude $\quad |A| = 3$

Period $\quad p = \dfrac{2\pi}{2} = \pi$

Phase Shift

$2x - \pi = 0 \quad$ to $\quad 2x - \pi = 2\pi$

$x = \dfrac{\pi}{2} \qquad\qquad x = \dfrac{3\pi}{2}$

4.2 - Translations of Sine and Cosine

Standard Form Equation

$$y = A \sin \left[B \left(x \pm \frac{C}{B} \right) \right]$$

Amplitude $|A|$

Period $p = \dfrac{2\pi}{B}$

Phase Shift $\dfrac{C}{B}$

4.2 - Translations of Sine and Cosine

Graphing Function $y = A\sin(Bx \pm C)$

Graph $y = 3\sin[-\frac{1}{2}(x - \pi)]$ $\longrightarrow$ $y = -3\sin[\frac{1}{2}(x - \pi)]$
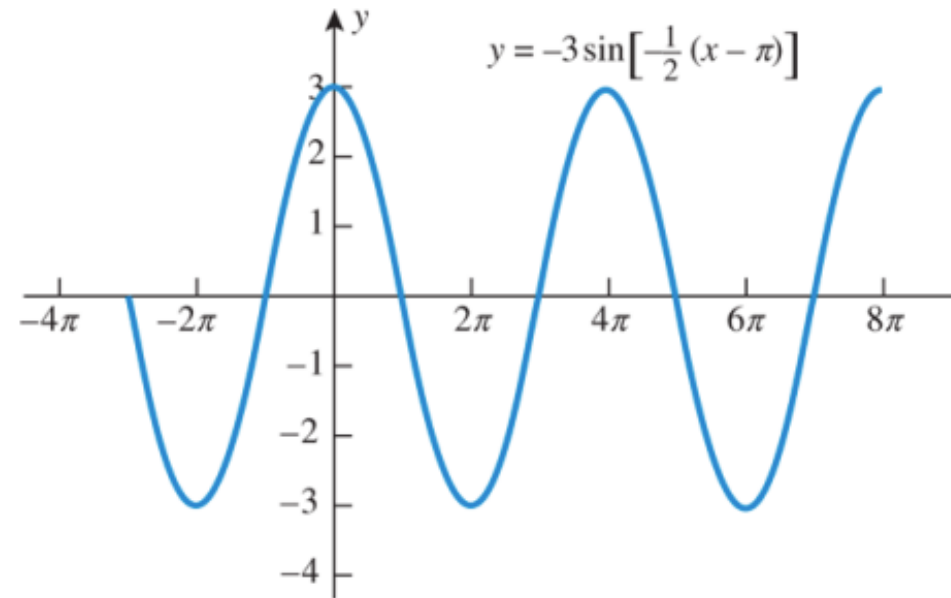
Amplitude $\quad |A| = 3$

Period $\quad p = \dfrac{2\pi}{\frac{1}{2}} = 4\pi$

Phase Shift

$\frac{1}{2}(x - \pi) = 0$ to $\frac{1}{2}(x - \pi) = 2\pi$

$x = \pi$ $\qquad\qquad x = 5\pi$

$y = -3\sin\left[-\frac{1}{2}(x - \pi)\right]$

4.2 - Translations of Sine and Cosine

Graphing Function $y = A \sin(Bx \pm C)$

Graph $y = -4 \sin(-2x + \pi) \longrightarrow y = 4 \sin[2(x - \frac{1}{2}\pi)]$
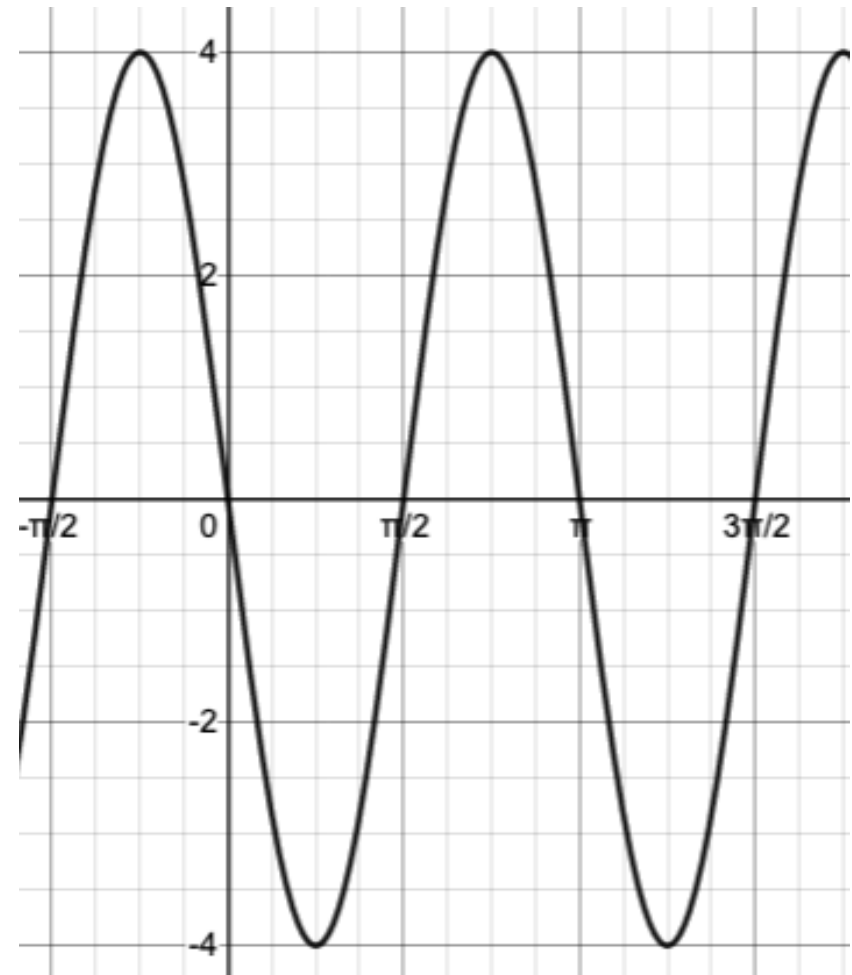
Amplitude $\quad |A| = 4$

Period $\quad p = \dfrac{2\pi}{2} = \pi$

Phase Shift

$\quad 2x - \pi = 0 \quad$ to $\quad 2x - \pi = 2\pi$

$\quad x = \dfrac{\pi}{2} \qquad\qquad x = \dfrac{3\pi}{2}$

4.2 - Translations of Sine and Cosine

Graphing Function $y = A \sin(Bx \pm C) + k$

Graph $y = 2\cos(2x - \pi) - 3$

Amplitude $|A| = 2$
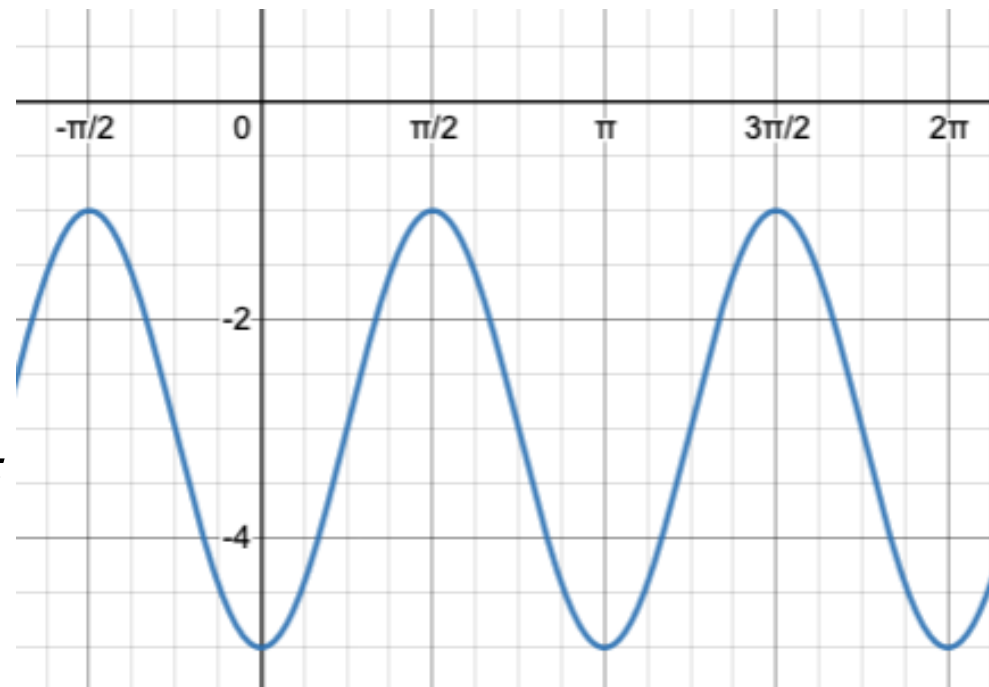
Period $p = \dfrac{2\pi}{2} = \pi$

Phase Shift

$2x - \pi = 0$ to $2x - \pi = 2\pi$

$x = \dfrac{\pi}{2}$ $x = \dfrac{3\pi}{2}$

4.2 - Translations of Sine and Cosine

Graphing Function $y = A\sin(Bx \pm C) + k$

Graph $y = -3\sin(2x + \pi) + 2$

Amplitude $\qquad |A| = 3$
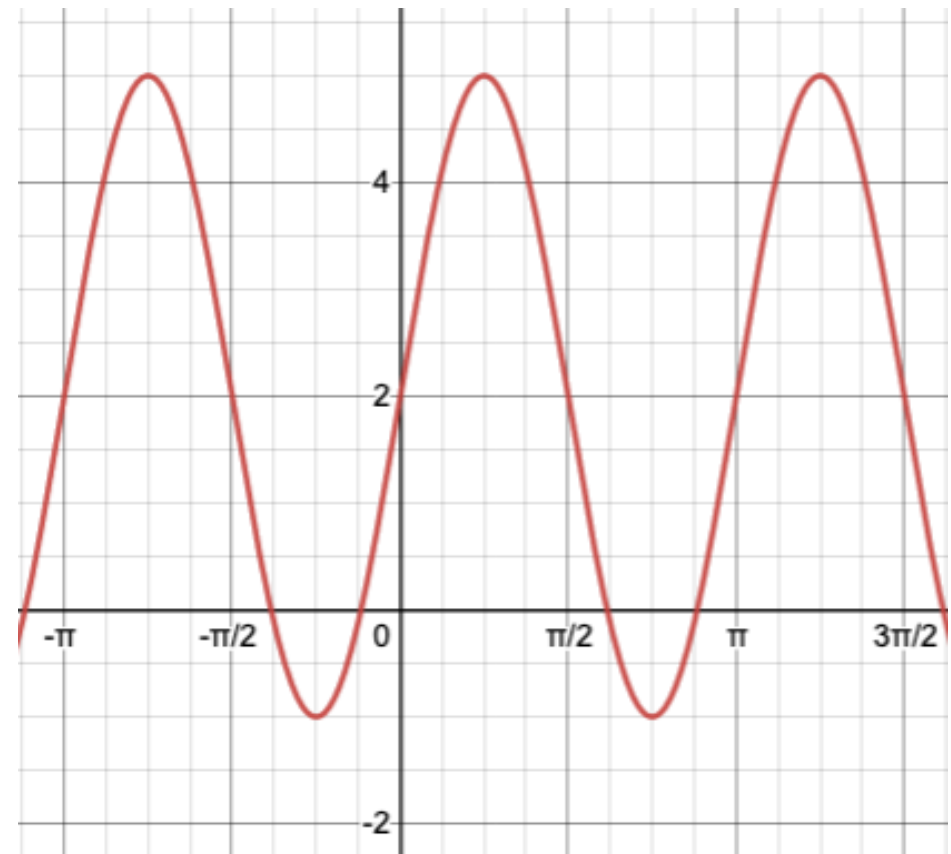
Period $\qquad p = \dfrac{2\pi}{2} = \pi$

Phase Shift

$2x + \pi = 0 \qquad$ to $\qquad 2x + \pi = 2\pi$

$x = -\dfrac{\pi}{2} \qquad\qquad\qquad x = \dfrac{\pi}{2}$

4.2 - Translations of Sine and Cosine

Constructing the Function $y = A\cos(Bx \pm C) + k$

Amplitude $\quad |A| = \left| \dfrac{max - min}{2} \right|$
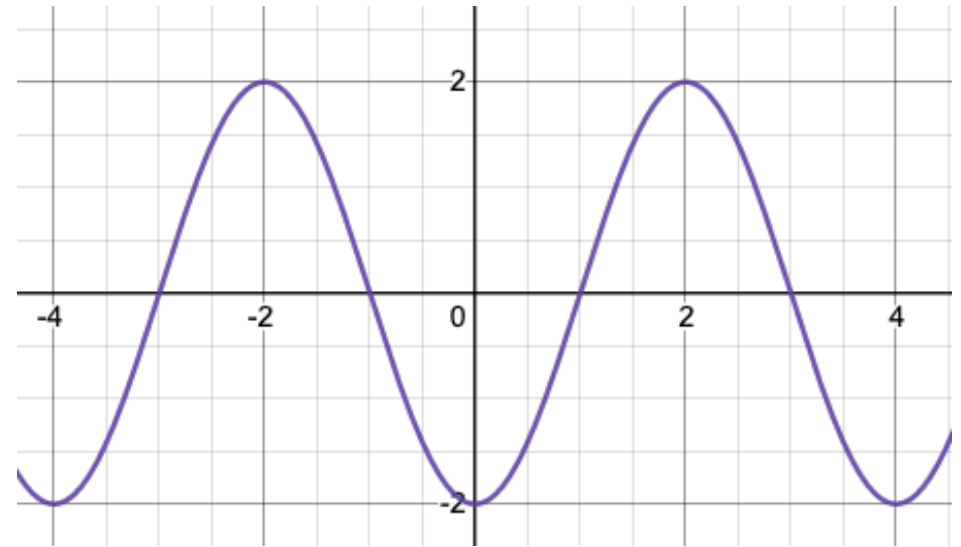
$$= \left| \dfrac{4}{2} \right| = 2$$

Period $\quad p = \dfrac{2\pi}{B} = 4$

$$B = \dfrac{\pi}{2}$$

Phase Shift (cos) $\quad -\dfrac{C}{B} = -\dfrac{C}{\frac{\pi}{2}} = 2 \quad C = -\pi$

Vertical Shift $\quad k = 0$

$$y = 2\cos\left(\dfrac{\pi}{2}x - \pi\right)$$
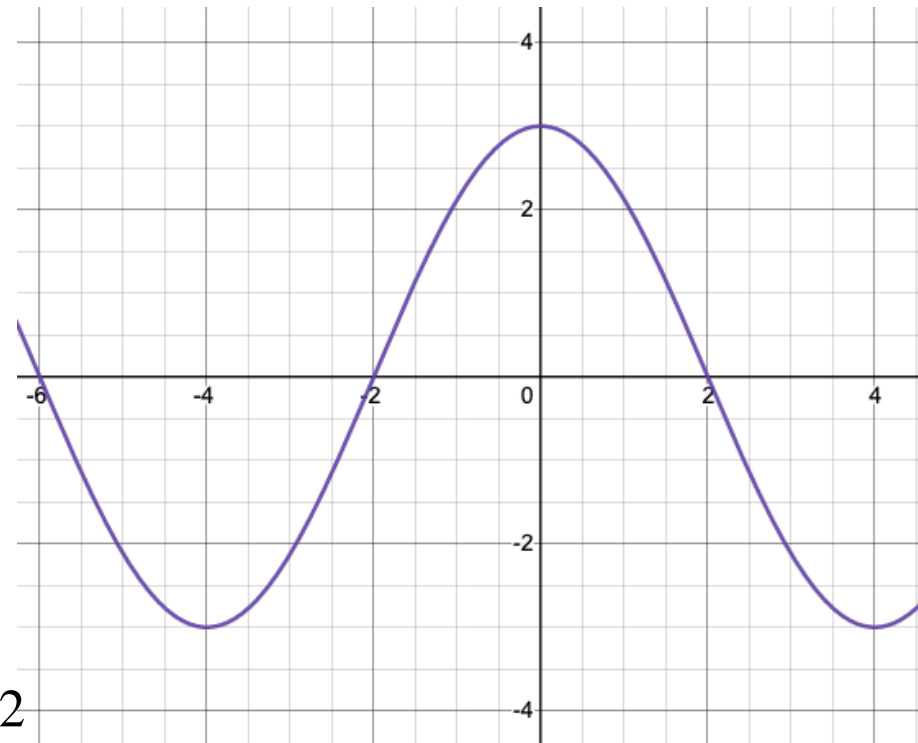
4.2 - Translations of Sine and Cosine

Constructing the Function $y = A\sin(Bx \pm C) + k$

Amplitude $\quad |A| = \left| \dfrac{max - min}{2} \right|$

$$= \left| \dfrac{6}{2} \right| = 3$$

Period $\qquad p = \dfrac{2\pi}{B} = 8$

$$B = \dfrac{\pi}{4}$$

Phase Shift (sin) $\quad -\dfrac{C}{B} = -\dfrac{C}{\frac{\pi}{4}} = -2$

$$C = \dfrac{\pi}{2}$$

Vertical Shift $\qquad k = 0$

$$y = 3\sin(\dfrac{\pi}{4}x + \dfrac{\pi}{2})$$